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# Korea's Growth Experience and Long-Term Growth Model

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#### Abstract

This paper analyzes the Republic of Korea's rapid and sustained growth experience for the past six decades from the perspective of the neoclassical growth model (the workhorse model of the World Bank's Long Term Growth Model (LTGM) project). Overall, the sources of Korea's growth were balanced among labor market and demographic factors, capital investment, human capital accumulation, and productivity growth. However, the main engine of growth evolved sequentially, e.g., labor and human capital factors in the 1960s, capital deepening in the 1970s, and then productivity growth for the following periods. The major sources of sustained growth over six decades were human capital accumulation and productivity growth rather than labor or capital investment. A counterfactual calibration of the model explains Korea's actual growth experience well, and shows why gaps between the model's predictions and the data arise. This illustrates that an appropriate calibration of a simple neoclassical growth model provides useful lessons and tools for policy makers in developing countries in designing their national development strategies.

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## Korea's Growth Experience and Long-Term Growth Model

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## 1 Introduction

A casual observer of the Republic of Korea's remarkable development experience, which Lucas (1993) indeed called a "miracle," is often impressed by its rapid and compressed growth experience but often overlooks two important features of Korea's development process: (i) how much adverse Korea's initial conditions were and (ii) the sustainability, not just the speed, of growth which has continued for about 60 years, overcoming various kinds of adverse initial conditions. In fact, this is exactly why Korea's development experience is valuable for other developing countries.

During the colonial era, Korea's cultural heritage and various kinds of the autonomous initiatives of development were adversely affected, which in turn resulted in the suppression of human capital formation, entrepreneurship and its own capacity of nation building. After the end of the colonial era, Korea again suffered from internal and external ideology fights, leading to a massive civil war, the Korean War, for three years.<sup>1</sup> After the end of the nationwide civil war, Korea was divided into two regions, which has been limiting the scope of economies of scale for national development. Korea tried to recover from the scar of the war and to reconstruct itself. (Hereafter, we will simply refer to the Republic of Korea as "Korea".) However, various kinds of corruption and disorder prevailed in Korea, which gave an excuse for the military to intervene in politics and a military coup overturned the government, followed by a series of political turmoils. In sum, the list of Korea's initial conditions includes almost all sorts of barriers to development such as colonial experience, civil war, corruption, a lack of physical and human resources, and political instability, which are recognized as major hurdles to development for most of the current developing countries. Korea was truly a devastated and poor nation when it first engaged in taking off to the path of miraculous growth, unaware of what would be coming.

Not all developing countries could achieve such performance of development after the end of the Second World War when many developing nations became independent. Global environments

<sup>&</sup>lt;sup>1</sup>The entire peninsula of Korea was the war field during the Korean War, which resulted 5.1 million casualties, 16% of residential structures and 40% of manufacturing factories and equipment were destroyed in the Republic of Korea, and 74% of electricity facilities, 89% of energy factories, and 70% of chemical factories were damaged in the Democratic People's Republic of Korea, when the War ended in 1953.

have changed and each developing country faces different kinds of challenges and development goals in the context of its own history. Thus, Korea's development experience per se would not be of any help for current developing countries. However, understanding the underlying mechanisms of such successful growth of Korea would be useful. This paper is an attempt to contribute to such understandings in the context of World Bank's recent initiative of the Long-Term Growth Model (LTGM) project.<sup>2</sup>

The World Bank's LTGM project aims to help the policy makers of developing countries design their national macroeconomic development policies from the perspective of the neoclassical growth model. By predicting the future growth paths stemming from the desired changes of investment and/or labor market policies such as the promotion of labor force participation, policy makers can better envision and quantify their development goals. This kind of quantitative policy design would be a great help in articulating their policy goals and also in materializing the actual changes. Furthermore, an explicit use of a structural growth model in doing this kind of quantitative exercises is clearly beneficial. At the same time, however, calibration of the structural model is always a challenge, particularly for prediction purposes in response to policy changes. Therefore, it would be useful to see if such an exercise can in fact be applied to a previous development experience for a country which already achieved the development goals that current developing countries are aiming for now. In this sense, the results of the application of the LTGM to Korea's development experience would deliver useful messages for other developing countries. This is the goal of this paper.

We first describe the canonical neoclassical growth model, which is the basis of the LTGM, as our accounting framework of this paper in Section 2. This model will be applied to Korea's economic growth for the 1960-2014 period to identify the underlying sources of Korea's GDP per capita growth in Section 3 by implementing a counterfactual decomposition analysis. Based on the analysis of Section 3, we calibrate Korea's economic growth in two ways to evaluate the performance of the LTGM in Section 4. First, we use the model as a tool for simulating Korea's

<sup>&</sup>lt;sup>2</sup>The LTGM is an Excel-based tool that allows users to simulate future long-term growth for most of the world's developing and emerging economies, building on the neoclassical growth model. See Pennings (2017) for a model description and contact LTGM@worldbank.org or Steven Pennings (spennings@worldbank.org) for further information. The LTGM builds on earlier work by Hevia and Loayza (2012).

growth process and evaluate the fit depending on the period of simulation as well as the policy dynamics. Second, we evaluate the model as a prescriptive tool to identify the influences of various sources of growth via the lens of Korea's 55 years of growth experience. Both types of calibration exercises illuminate the nature of the LTGM in analyzing the future growth process of developing countries. Section 5 concludes.

## 2 Neoclassical Growth Model as an Accounting Framework

We consider the standard neoclassical growth model based on the aggregate production function, which was first proposed by Solow (1956) postulating the relationship between inputs and output at aggregate levels such that

$$Y_t = A_t X_t \tag{1}$$

where  $Y_t$  denotes the aggregate output,  $X_t$  the composite input (sometimes referred as "total factor"), and  $A_t$  the total factor productivity (TFP) at period t. The composite input  $X_t$  consists of capital  $K_t$  and effective unit of labor  $\widetilde{L}_t$  such that

$$X_t = F\left(K_t, \widetilde{L}_t\right).$$

where the production function satisfies the canonical properties of the neoclassical growth model, i.e., (i) monotonicity, (ii) diminishing returns and (iii) constant returns to scale.<sup>3</sup>

The capital is accumulated according to the following law of motion

$$K_{t+1} = I_t + (1 - \delta) K_t, \tag{2}$$

where  $I_t$  denotes the capital investment and  $\delta$  the depreciation rate of existing capital stock. The diminishing returns property is the key property of the neoclassical growth model which

<sup>&</sup>lt;sup>3</sup>The property of "monotonicity" means that addition of capital and labor contributes to increasing output (i.e.,  $\frac{\partial F}{\partial K} \geq 0$  and  $\frac{\partial F}{\partial L} \geq 0$  for all K and L). The property of "diminishing returns" means that the marginal contribution of adding more inputs decreases as the amount of inputs increases (i.e.,  $\frac{\partial}{\partial K} \left(\frac{\partial F}{\partial K}\right) < 0$  and  $\frac{\partial}{\partial L} \left(\frac{\partial F}{\partial L}\right) < 0$ ). The property of "constant returns to scale" means proportional changes in all inputs at the same time induces the same proportion of changes in output (i.e.,  $F\left(cK_t, c\widetilde{L_t}\right) = cF\left(K_t, \widetilde{L_t}\right)$  for all c > 0).

stabilizes the growth dynamics to exogenous shocks. Owing to this property, the incremental capital decreases over time unless there exists strong enough growth in TFP.

We can further decompose the effective unit of labor into human capital per worker  $h_t$  and the employment size  $L_t$  (measured by the number of workers) such that  $\widetilde{L}_t = h_t L_t$ , in other words  $X_t = F(K_t, h_t L_t)$ 

There is no direct data for the TFP variable, hence it is typically measured by the residual such that

$$A_t = \frac{Y_t}{F\left(K_t, h_t L_t\right)}.\tag{3}$$

We do not need to assume any functional form on the production function F to perform the standard growth accounting. However, to facilitate the measurement of the level of the TFP, we have to choose a functional form for F. The most standard functional form for the aggregate production function is the Cobb-Douglas form such as

$$Y_t = K_t^{1-\beta} \left( A_{L,t} h_t L_t \right)^{\beta},$$
(4)

where the only parameter  $\beta$  corresponds to the labor share in national income account, and  $A_{L,t}$  denotes the labor-augmenting technology level. This can be re-expressed such that

$$Y_t = A_t X_t,$$
  

$$A_t = A_{L,t}^{\beta},$$
  

$$X_t = K_t^{1-\beta} (h_t L_t)^{\beta},$$
(5)

which has the same form of representation as in equation (1). In per worker terms, we can also re-express the Cobb-Douglas production function such that

$$y_t = A_t k_t^{1-\beta} h_t^{\beta} \tag{6}$$

where  $y_t = Y_t/L_t$  and  $k_t = K_t/L_t$ . Another way to represent the output per worker is

$$y_t = A_{L,t} \left( K_t / Y_t \right)^{\frac{1-\beta}{\beta}} h_t.$$
(7)

From equation (6), we can derive the following (and typical) growth accounting formula

$$\widehat{y}_t = \widehat{A}_t + (1 - \beta)\,\widehat{k}_t + \beta\widehat{h}_t,\tag{8}$$

where the "hat" notation denotes the growth rate of the corresponding variable, e.g.,  $\hat{y}_t \equiv \frac{dy_t/dt}{y_t}$ . From equation (7), we can derive another growth accounting formula

$$\widehat{y}_t = \widehat{A_{Lt}} + \left(\frac{1-\beta}{\beta}\right)\left(\overline{\left(\frac{K}{Y}\right)}_t + \widehat{h}_t.$$
(9)

From equation (5), the typical TFP growth rate  $\widehat{A}_t$  is related to the labor-augmenting productivity growth  $\widehat{A}_{Lt}$  as follows

$$\widehat{A}_t = \beta \widehat{A}_{Lt}.$$
(10)

The equations (8) and (9) show two different ways of expressing the growth accounting of output per worker. Each formula serves its own purpose of decomposing the growth of output per worker. Equation (8) is to be used when we want to quantify the contributions of each factor and the TFP to the growth of output per worker at surface level. However, this formula has a limitation. The growth of capital per worker can be induced by the growth of the TFP because the increase in TFP raises the marginal product of capital. Therefore, the observed growth of capital is an outcome of two effects, first the pure investment effect and second the TFP-induced effect. If we want to isolate the genuine contribution of capital accumulation purely from investment, we should use the second formula, which decomposes the growth of output per worker into pure productivity growth effect  $\widehat{A}_{Lt}$ , human capital growth effect  $\widehat{h}_t$ , and capital-deepening effect  $\left(\frac{1-\beta}{\beta}\right) \widehat{\left(\frac{K}{Y}\right)_t}$ . The capital-deepening effect isolates the genuine capital accumulation effect because the increase in productivity would directly raise the output but also the capital owing to the increase in marginal product of capital. Thus, the capital-output ratio  $\frac{K}{Y}$  increases, this would capture the genuine effect of capital growth due to the capital investment. This is the intuition behind considering the capital-deepening effect  $\left(\frac{1-\beta}{\beta}\right) \left(\frac{K}{Y}\right)_t$  as the genuine capital accumulation effect. We will use this version of growth accounting formula as our benchmark framework in accounting for the growth of output per worker.

The conventional measure of the level of development or national welfare is the GDP per capita  $y_{P,t} \equiv Y_t/N_t$  (where  $N_t$  is the total population size) rather than the GDP per worker  $y_t \equiv Y_t/L_t$  above. GDP per capita differs from GDP per worker by the two demographic features of the labor market, (i) the labor force participation rate  $S_{E,t} \equiv L_t/N_{L,t}$  and (ii) the working-age population share  $S_{W,t} \equiv N_{L,t}/N_t$ , where  $N_{L,t}$  is the working-age population (age group of 15-64) size, and  $L_t$  is the labor force size such that

$$y_{P,t} = S_{W,t} S_{E,t} y_t, \tag{11}$$

and in growth terms

$$\widehat{y_{P,t}} = \widehat{S_{W,t}} + \widehat{S_{E,t}} + \widehat{y_t}.^4 \tag{12}$$

## 3 Analysis of Korea's Economic Growth<sup>5</sup>

#### 3.1 Data

Equations (9) and (12) are our framework accounting for Korea's economic growth (which can be used for analyzing any country's growth). To quantify these equations, we need the following data series for our sample period 1960-2014, and their sources are in brackets as follows: (1) total population size [World Development Indicators (WDI)] for  $N_t$ , (2) working-age population share [WDI] for  $S_{W,t}$ , (3) labor force participation rate [WDI] for  $S_{E,,t}$ , (4) real GDP at constant 2011 national prices (in 2011 million US\$) ["rgdpna" in Penn World Table version 9.0 (PWT 9.0)] for  $Y_t$ , (5) capital stock at constant 2011 national prices (in 2011 million US\$) ["rkna" in PWT 9.0] for  $K_t$ , (6) human capital per worker ["hc" in PWT 9.0] for  $h_t$ , (7) labor force size [WDI] for  $L_t$ , (8) labor share ["labsh" in PWT 9.0] for  $\beta$ , (9) capital depreciation rate ["delta" in PWT 9.0] for  $\delta$ , (10) labor-augmenting technology level [calculated from equation (4)], and (11) investment [calculated using investment rate data "csh\_i" from PWT 9.0].<sup>6</sup> The value of the average labor share which we calibrate for the parameter  $\beta$  is 0.602. The value of the average depreciation rate which we calibrate for the parameter  $\delta$  is 0.053.

<sup>5</sup>Part of the analysis of this section is based on Jeong (2016).

<sup>6</sup>Original data source of the WDI labor variables such as working-age population, labor force participation rate is the International Labor Organization (ILO) Statistics. The labor share and the capital depreciation rate variables are time-varying in PWT 9.0 and we take the time-series averages during our sample period 1960-2014.

<sup>&</sup>lt;sup>4</sup>We use labor force data from WDI for  $L_t$  to maintain the consistency with the data use protocol of the LTGM project so that there are possible differences in labor force participation rate between the national sources and the WDI. Furthermore, using labor force instead of employment data may generate the different growth rate of  $\widehat{S}_{W,t}$ . However, using the national source data, we find that labor force participation rate and employment rate tightly co-move with each other and the growth rates of  $\widehat{S}_{W,t}$  between the two measures differ only by 0.1% for the sample period.

#### 3.2 Features of Korean Economic Growth

Measuring the size of the Korean economy by the total GDP, Korean economy's size increased by 44 times for the 1960-2014 period. Our measure of GDP is the real GDP at constant 2011 national prices in 2011 million US\$ value. Measuring the individual standard of living by the GDP per capita, Korea's standard of living increased by 22 times from \$1,557 in 1960 to \$34,300 in 2014, implying the annual average growth rate of 6% for about two generations. Thus, Korea's economic growth has been not only rapid but also sustained. The rapid growth of the Korean economy is well-known. However, the sustaining feature of Korean growth is rather less so. The labor productivity, measured by the GDP per worker, grew by 4.8% each year on average.

Figure 1 plots the path of Korea's annual growth rate of the GDP per capita with the quartic-fit time trend, which shows a hump-shaped trend.<sup>7</sup> That is, the growth of GDP per capita accelerated for the 1960-1980 period and then gradually slowed down afterwards. There are two noticeable dips in Figure 2: the first one at 1980 and the second one at 1998. These were the only episodes of negative growth during Korea's development experience after the year 1960. There was a serious cold weather shock combined with the political turmoil during the 1979-1980 period, and the well-known Asian financial crisis during the 1997-1998 period. At the same time, Figure 1 also shows that the recovery from those adverse shocks was very fast in Korea.

Figure 2 plots the path of Korea's GDP per capita and that of GDP per worker, which shows that the growth of Korea's GDP per capita is mainly driven by the GDP per worker. There are three sources of GDP per worker growth as in equation (7): (1) capital-output ratio, (2) human capital per worker and (3) labor-augmenting technology (which is linked to TFP as in equation (5)). Figures 3 to 5 display the growth of these three sources of growth of GDP per worker, respectively.

Figure 3 plots the capital-output ratio ("K/Y") and the investment rate ("IR") together with their quartic-fit trends. This figure shows that Korea's capital-output ratio almost monotonically increased from 1.31 in 1960 to 3.88 in 2009, and then slightly decreased to 3.85 in 2014. The

<sup>&</sup>lt;sup>7</sup>By the "quartic-fit time trend," we mean the fitting regression line of the GDP per capita against the fourth-order polynomial function.











average investment rate for the entire sample period is 31.5%. However, the investment rate was not constant over time. It first increased sharply from 11.7% to 43% in 1979, and then gradually decreased to 30.4% by 2014 with fluctuation. The sharp rise of Korea's investment rate in the 1970's was mainly driven by the industrial policies in relation to export promotion and establishment of heavy-and-chemical industries.

The human capital per worker (which is the rate-of-return weighted total years of schooling index) monotonically increased throughout the sample period at the annual average growth rate of 1.52%, but in a concave way, i.e. the growth rate of human capital has decreased over time from



2.7% in 1960 to 0.9% in 2014, as is shown in Figure 4. Figure 5 shows that the labor-augmenting technology level (what we would call "productivity" which has a one-to-one relationship with the TFP) also almost monotonically increased by 2.8 times, implying the annual average growth rate of 1.91%. Unlike the human capital growth, the path of the productivity growth rate does not show much salient trends. It is just mildly hump-shaped.

There are two labor market demographic factors to the GDP per capita growth other than the GDP per worker growth, i.e., the changes of working-age population share and those of labor force participation rate, which are displayed in Figures 6 and 7.









Figure 7: Korea's Labor Force Participation Rate



The working-age population share was stagnant in the 1960s but increased rapidly in the 1970s and 1980s, and then the increasing speed slowed down until 2014. Overall Korea's working-age population share increased from 55% in 1960 to 73% in 2014. The labor force participation rate also increased during the sample period from 52% in 1960 to 68% in 2014.<sup>8</sup>, <sup>9</sup> Thus, the changes in these labor market demographic factors positively contributed to growth of the GDP per capita as shown in equation (12).

#### 3.3 Decomposition Analysis

Applying our accounting framework of equations (9) and (12) to the above data, we can decompose Korea's growth of GDP per capita for the 1960-2014 period by constructing counterfactual GDP per capita measures as follows. Combining equations (7) and (11), we express the GDP per capita such that

$$y_{P,t} = S_{W,t} S_{E,t} A_{L,t} \left( K_t / Y_t \right)^{\frac{1-\beta}{\beta}} h_t.$$
(13)

In order to isolate the contribution of productivity growth to GDP per capita growth, we fix the values capital-output ratio, human capital per worker, working-age population share and labor force participation rate at the 1960 values and vary only the labor-augmenting technology level as in the data. That is, the counterfactual GDP per capita measure due to the productivity changes is

$$y_{P,t}^{A_L} = S_{W,1960} S_{E,,1960} A_{L,t} \left( K_{1960} / Y_{1960} \right)^{\frac{1-\beta}{\beta}} h_{1960}$$

<sup>&</sup>lt;sup>8</sup>Not surprisingly, this increase of labor force participation rate was due to the rise of the female labor force participation rate from 39% in 1960 to 57% in 2014. The male labor force participation rate increased from 75% in 1960 only to 79% in 2014. Even in the year of 2014, there still exists a substantial gap in labor force participation rate between men and women, although the gap dropped significantly since 1960.

<sup>&</sup>lt;sup>9</sup>We observe a noticeable up and down in labor force participation rate between the mid-1970s and early 1980s. Using the national data sources of population census and labor force survey data from the Korean Statistical Information Service (KOSIS), we find that this is an outcome of the combination of the WDI data issue and the reality of the Korean economy. The WDI labor force data for Korea for the mid-1970s is a little overestimated, which generates the more rapid rise of the labor force participation rate than the trend. (It seems to happen because the WDI working-age population and labor force data are based on the estimates from the UN population project and ILO, which can be different from the ex-post national census and surveys.) However, the fall in the labor force participation rate (as well as in the employment rate) for the early 1980s reflects the actual recession of the Korean economy. In 1979, President Park was assassinated and there was a military coup in the following year, which generated economic instability. Furthermore, there was unprecedented cold-weather damage in 1980.

and the growth rate of this counterfactual measure is

$$\widehat{y_{P,t}^{A_L}} = \widehat{A_{L,t}}.$$

We can similarly construct counterfactual measures of GDP per capita due to the changes of other components. Figure 8 plots those counterfactual GDP per capita measures for each of the five components of productivity (labeled as "AL"), human capital per worker (labeled as "HC"), capital deepening (labeled as "K/Y"), working-age population share (labeled as "WAP"), and labor force participation rate (labeled as "LFP"). Table 1 summarizes the growth rates of the actual and the above counterfactual measures of GDP per capita for the entire period as well as for each of the sub-period by decade (1960s, 1970s, 1980s, 1990s and 2000s) and the remaining 2010-2014 period.

Figure 8 and Table 1 reveal some interesting features of Korea's economic growth, which have not been recognized before. First, the largest contributing component is productivity growth (1.9% each year on average) rather than factor growth during the entire sample period. The second largest contributing component is human capital growth (1.5% each year on average), and then the third one is the capital deepening effect (1.3% each year on average). Second, despite such contribution ordering, the magnitudes of contribution are all substantial for each component, and they are more or less similar among these three components. That is, there has been no single dominant component during the process of Korea's economic growth. Furthermore, the contribution of the two labor market demographic factors, increases in working-age population share and labor force participation rate combined, to increasing Korea's GDP per capita by 1.0% each year on average, which is not a small magnitude.

Comparing the contributions of the five growth components across sub-periods, major contributing components are different over time. In the initial development stage of the 1960's, human capital growth was the major driving force of Korea's growth, increasing GDP per capita by 2.2% each year on average. However, the human capital growth effect gradually but monotonically decreases afterwards. In the 1970's, however, capital deepening was the main source of growth, contributing to GDP per capita growth by 3% each year on average. The capital deepening effect dropped remarkably to 0.8% in the 1980s, surging back to 1.9% in the 1990s,



Note (1) Each line represents the counterfactual path of GDP per capita from the isolated growth of each variable.

Note (2) "AL": Productivity growth of labor-augmenting technology, "HC": Human capital growth, "K/Y": Capital deepening, "WAP": Changes of working-age population share, "LFP": Changes of labor force participation rate.

Period	Total	WAP	LFP	AL	HC	K/Y	TFP
1960-2014	5.9	0.5	0.5	1.9	1.5	1.3	1.1
1960-1970	5.0	-0.1	1.2	0.8	2.2	1.0	0.5
1970-1980	7.4	1.3	-0.3	1.2	1.9	3.0	0.7
1980-1990	8.6	1.1	1.1	3.7	1.7	0.8	2.2
1990-2000	6.0	0.3	0.2	2.3	1.2	1.9	1.4
2000-2010	3.9	0.1	0.2	2.2	0.8	0.5	1.3
2010-2014	2.5	0.1	0.8	0.5	0.9	0.3	0.3

Table 1: Decomposition of Sources of Korea's Growth of GDP per Capita (%)

Note (1) Each column represents the contribution of each variable to the annual average growth rate of GDP per capita.

Note (2) "Total": Total growth of GDP per capita, "WAP": Contribution of changes of working-age population share, "LFP": Contribution of changes of labor force participation rate, "AL": Contribution of productivity growth of labor-augmenting technology, "HC": Contribution of human capital accumulation, "K/Y": Contribution of capital deepening, "TFP": Contribution of total factor productivity (which is equal to the labor share times "AL") and then diminished to 0.5% for the 2000s period and further to 0.3% for the 2010-2014 period.

For the rest of sub-periods of the 1980s, 1990s and 2000s, productivity growth was the main engine of Korea's growth, neither the capital deepening nor the human capital growth. In the 1980s, productivity growth only increased the GDP per capita by 3.7% per year on average.<sup>10</sup> The contribution shares of the productivity growth out of the total growth of the GDP per capita were 43%, 38% and 56% during the 1980s, 1990s, and 2000s, respectively. These contrasting findings of the changing contributions between productivity growth and factor growth together with the decreasing magnitudes of human and physical capital growth may signal that the forces of diminishing returns have become stronger in the process of Korea's economic growth.

### 4 Calibration of Korea's Economic Growth

### 4.1 LTGM of the World Bank

The neoclassical growth model that we used as an accounting framework in analyzing Korea's economic growth can also be used as a simulation device for the future growth if we can make a reasonable conjecture about the parameter values of the model that will govern in the future. The other way of using the same model is to make inferences about the future policies regarding the parameter values that are needed to reach the pre-set growth goal in the future. This way of utilizing the neoclassical growth model is recently labeled as the "Long-Term Growth Model (LTGM)" approach by the World Bank for the purpose of helping policy makers in developing countries to design their macroeconomic growth policies.

In terms of contents of the model, the World Bank's basic LTGM is just the same as the neoclassical growth model in Section 2. How to use such model for prediction or policy design purposes depends on how to calibrate the model. This kind of calibration is not an easy exercise because we need to calibrate the model to fit the future that we do not observe at the moment of calibration. The analysis of Korea's economic growth in Section 3 can be utilized in finding the right ways of calibrating the model in the following sense. Suppose there were policy makers

<sup>&</sup>lt;sup>10</sup>For the purpose of comparison of this productivity growth measure with the standard TFP growth, the last column of Table 1 reports the standard TFP growth implied by our productivity growth measure as in equation (10).

in the past in Korea, say in 1970, who wanted to predict what would happen to GDP per capita growth after 1970 and the only available information set was the data for the 1960-1970 period. Then, we may ask what would be the best way for them to calibrate the underlying parameters of the model. We can answer this question because unlike the fictitious policy makers in 1970, we in fact know what actually happened after 1970 in Korea so that we can evaluate the calibration method by evaluating the prediction performance against the actual data.

#### 4.2 Objects of Calibration

We first need to determine the set of parameters to calibrate. The GDP per capita at period t is as in equation (13)

$$y_{P,t} = S_{W,t} S_{E,t} A_{L,t} \left( K_t / Y_t \right)^{\frac{1-\beta}{\beta}} h_t$$

and the gross growth rate of the GDP per capita between period t and t + 1 is

$$\frac{y_{P,t+1}}{y_{P,t}} = \Lambda_{t+1}^{\beta} \left[ \frac{\gamma_t \frac{Y_t}{K_t} + (1-\delta)}{1+\hat{N}_{t+1}} \right]^{1-\beta}$$
(14)

where

$$\Lambda_{t+1} = \left(1 + \widehat{S}_{W,t+1}\right) \left(1 + \widehat{S}_{E,t+1}\right) \left(1 + \widehat{A}_{L,t+1}\right) \left(1 + \widehat{h}_{t+1}\right),$$

 $\gamma_t$  is the investment rate at period t, and  $\hat{N}_{t+1}$ ,  $\hat{S}_{W,t+1}$ ,  $\hat{S}_{E,t+1}$ ,  $\hat{A}_{L,t+1}$ , and  $\hat{h}_{t+1}$  are the growth rates of population, working-age population share, labor force participation rate, productivity, and human capital between periods t and t + 1, respectively. The growth equation (14) clarifies two things. First, the growth rate of GDP per capita increases in investment rate  $\gamma_t$ , but this growth effect decreases in  $K_t/Y_t$ , i.e., the capital-output ratio of the base year. The latter decreasing growth effect from investment captures the diminishing returns property of the neoclassical growth model. Second, it increases in growth rates of working-age population, labor force participation rate, productivity, and human capital but decreases in population growth rate.

Now in order to simulate the growth path using equation (14), we need to select the parameters  $(1 - \beta, \delta)$  and to calibrate the growth rates of  $\widehat{N}_{t+1}$ ,  $\widehat{S}_{W,t+1}$ ,  $\widehat{S}_{E,t+1}$ ,  $\widehat{A}_{L,t+1}$ , and  $\widehat{h}_{t+1}$ . When we substitute these growth rates with the actual data, we will get the precise growth rate. For the purpose of simulation, we should choose a way to calibrate the growth rates of these five growth variables at period t + 1 as well as the time-invariant parameters  $1 - \beta$  and  $\delta$  from the observed data. Furthermore, to apply growth equation (14) to the next period at period t+2, we need to calibrate  $\gamma_{t+1}$  also. Typical neoclassical growth models assume that  $\widehat{A}_{L,t+1}$  and  $\widehat{N}_{t+1}$  are constant for all periods (we may make similar assumption on  $\widehat{h}_{t+1}$ ), but they are silent about the changing rates of  $\gamma_{t+1}$ ,  $\widehat{S}_{W,t+1}$ ,  $\widehat{S}_{E,t+1}$  and  $\widehat{h}_{t+1}$ . In fact, we cannot make such nonzero constant growth assumption for  $\gamma_{t+1}$ ,  $\widehat{S}_{W,t+1}$  and  $\widehat{S}_{E,t+1}$  because they are "share" variables which are upper-bounded. Thus, we need to choose a way to predict the path for  $\gamma_{t+1}$ ,  $S_{W,t+1}$ and  $S_{E,t+1}$  during the targeted future period for the simulation purpose. Furthermore, these three variables are labeled as "time-varying policy parameters" which would change depending on demographics and policies.

#### 4.3 Calibration 1: Status-quo Simulation Approach

To evaluate the neoclassical growth model as a simulation tool as the World Bank's LTGM project does, we would like to vary the calibration method and compare the patterns as well as the performance of the prediction of the model to seek the best way to choose the calibration objects, i.e., the future growth rates  $(\hat{N}_{t+1}, \hat{A}_{L,t+1}, \hat{h}_{t+1})$  and the time-varying policy parameters  $(\gamma_{t+1}, S_{W,t+1}, S_{E,t+1})$ , in order to simulate the growth path of GDP per capita. Regarding the labor share and the depreciation rate parameters, we will fix them at the same values as in the decomposition analysis of the actual Korean economy in Section 3 in order to isolate the effects of the calibration method on simulation.<sup>11</sup>

The first and the most straightforward way of calibration is to simply follow the canonical neoclassical growth model, where the productivity and population grow at constant rates  $\widehat{A}_{L,t+1} = g_{A_L}$ ,  $\widehat{N}_{t+1} = g_N$  for all periods. We may make similar constant growth rate assumption for the human capital as well such that  $\widehat{h}_{t+1} = g_h$  for all periods. The canonical neoclassical growth model also assume that investment rate is constant such that  $\gamma_{t+1} = \gamma_t = \gamma_0$ . This assumption of "constant rates" in fact can be a reasonable one when the economy is near the steady state and the economy grows close to the balanced growth path, along which the growth rates are determined mainly by the fundamental parameters of technology and preferences. Consistent

<sup>&</sup>lt;sup>11</sup>That is,  $1 - \alpha = 0.602$  and  $\delta = 0.053$ .

way of calibrating the labor market demographic factors with this "steady-state assumption" is to choose that  $S_{W,t+1} = S_{W,t} = S_{W,0}$  and  $S_{E,t+1} = S_{E,t} = S_{E,0}$  (so that  $\hat{S}_{W,t+1} = 0$  and  $\hat{S}_{E,t+1} = 0$ ) for all periods.

Suppose that a policy maker in Korea made this set of "steady-state assumptions" in 1970, and then applied the benchmark growth model to simulate the GDP per capita for the future period of 1971-2014. Suppose that the data available for this policy maker in 1970 are the 1960-1970 period data. Once deciding to take the "steady-state" approach, the best way to calibrate the constant growth rates of  $g_{A_L}$ ,  $g_h$ , and  $g_N$  would be to form an adaptive expectation and the best fits for the constant growth rate parameters would be the long-term average growth rates, represented by the annual average growth rates of the corresponding variables for the dataavailable period, i.e., the 1960-1970 period. In selecting the constant values for the investment rate, working-age population share, and labor force participation rate, we may want to take the average yalues for the past sample period to smooth out the shocks. However, if taking the averaging period too long, the average values would not represent the true values of the parameters for the simulation period. Thus, the average values for the initial five-year period prior to the starting date of simulation, for example, the 1966-1970 period values for the 1970 simulated prediction, are to be used to calibrate the investment rate, working-age population share, and labor force participation rate.

We can repeat the above simulation exercise by changing the starting year to 1980 (using the 1970-1980 data) or to 1990 (using the 1980-1990 data) instead of 1970 using the same calibration method. Comparison of the three sets of prediction results would be informative because Korean economy has evolved from a transition economy toward a steady-state economy. The calibrated values for the three sets of simulated prediction exercises, labeled as "Pred\_70", "Pred\_80", and "Pred\_90", respectively for the 1970, 1980, and 1990 simulation, by the above steady-state calibration method are summarized in Table 2. For the purpose of referencing with other countries, in Table 2, we also indicate the average purchasing-power-parity adjusted real GDP per capita level for each period when the parameter values of  $\gamma_0$ ,  $S_{W,0}$ . and  $S_{E,0}$  are chosen.<sup>12</sup> For example, Korea's average PPP-adjusted real income level was \$1,466 in 1960s

<sup>&</sup>lt;sup>12</sup>Note that this real income measure is obtained from the "rgdpe" in PWT 9.0 divided by the WDI population

rasie - campiacea i arameter (araes nom statas que inspireae	Table 2:	Calibrated	Parameter	Values	from	Status-qu	o Ap	proach
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Simulation	$g_A$	$g_h$	$g_N$	$\gamma_0$	$S_{W,0}$	$S_{E,0}$	PPP Real Income (2011 USD)
Pred_70	0.8%	2.2%	2.6%	0.27	0.54	0.56	1,466 (1960s)
Pred_80	1.2%	1.9%	1.7%	0.37	0.61	0.59	3,844 (1970s)
Pred_90	3.7%	1.7%	1.2%	0.35	0.68	0.61	7,688 (1980s)

Note.  $g_{A_L}$ : Annual growth rate of productivity of labor-augmenting technology,  $g_h$ : Annual growth rate of human capital per worker,  $g_N$ : Annual growth rate of population,  $\gamma_0$ : Investment rate,  $S_{W,0}$ : Working-age population share,  $S_{E,0}$ : Labor force participation rate.

when the investment rate was 0.27, working-age population share was 0.54 and the labor force participation rate was 0.56.

Figure 9 compares the predicted paths of GDP per capita of the three simulations (similarly labeled as in Table 2), overlaid with the actual path (labeled as "Actual"). This comparison illuminates important features of the LTGM as a simulated prediction device as follows.

First, notice that the "Pred\_70" simulation under-predicts the GDP per capita as shown in Figure 9. It fits only the very beginning-of-period data, i.e., for the 1971-1973 period. The prediction diverges away below the actual one afterwards. This result is not a surprise, reflecting Figures 3, 6, and 7, which show that the investment rate, working-age population share, and labor force participation rate all increased during the 1960s, hence the 5-year average values underestimate the future values. Furthermore, the investment rate and the working-age population share further increased in the 1970s compared to the 1960s values. The investment rate got stabilized after the early 1980s, and the increase of the working-age population share also slowed down after the 1990s. The labor force participation rate continues to show increasing trend except the substantial dip during the 1977-1986 period. Furthermore, Korea's population growth rate has fallen monotonically during the entire sample period from 3% in the 1960 to 0.4% in 2014. All these changes have increasing effects of GDP per capita, which are not captured by the current calibration method. Figure 4 illustrated the declining growth rate of human capital, particularly after the 1990s. Thus, current calibration method tends to over-estimate data, hence is different from our GDP per capita measure which is calculated from the "rgdpna" in PWT 9.0. In Table 2, we use the "rgdpe" measure to facilitate the cross-country comparison of development level.





Note (1) Each line represents the actual or the predicted path of GDP per capita at different starting date of simulation according to the LTGM using the prior data.

Note (2) "Actual": Actual GDP per capita, "Pred\_70": Predicted GDP per capita in the year 1970, "Pred\_80": Predicted GDP per capita in the year 1980, "Pred\_90": Predicted GDP per capita in the year 1990.

the GDP per capita after the 1990s and on. Figure 5 showed that the productivity growth rate has been more or less constant during the sample period. Thus, current calibration method is a reasonable one regarding productivity growth. In sum, the under-prediction of the Pred\_70 using the steady-state cum status-quo approach calibration method seems to be mainly due to the assumptions of the constant rates of investment, working-age population, and labor force participation. Observing the "Pred\_80" simulation, we get similar results, although the fitting performance improves over the "Pred\_70" simulation. In contrast, the 1990 prediction, which uses the 1980s data, fits the data very closely during the 17-year period (1991-2007), and then the model over-predicts the GDP per capita after 2008 with increasing gap. The main reason behind the good fit for the 1991-2007 period is that there were no clear trends for the investment rate, though being subject to fluctuations, so that the capital-deepening effects are well captured during this period. However, the over-prediction of the "Pred\_90" for the 2008-2014 period seems to be caused by various reasons: (i) the gradual slowdown of human capital accumulation, (ii) decreasing investment rate, particularly after 2005, (iii) the stagnation of working-age population share after 2000, (iv) the sudden stagnation of productivity after 2010, which can be confirmed by Table 1 and Figures 3 to 7. Comparing the above patterns of predictions across Pred\_70, Pred\_80, and Pred\_90, we learn that the prediction performance of the LTGM would be good when the economy grows in the stabilized environments, but the LTGM tends to under-predict when the parameters of investment rate, working-age population share, and labor force participation rate are actively changing. The LTGM may over-predict the growth when the economy is near the final phase of transitional growth (and/or subject to negative productivity shocks).

## 4.4 Calibration 2: Time-varying Parameter Embedded Simulation Approach

Another way of using the LTGM is to evaluate the expected changes of income growth in response to the different parameters of growth. For this exercise, we categorize the six parameters of calibration of the LTGM in the following manner. The rates of productivity growth and human capital growth are the determinants of the steady-state growth fundamentals. Thus, we call these two growth rates as "fundamental parameters." The changes of the rest of the

variables are related to transitional growth. The changes of working-age population share, labor force participation rate, and population growth rate affect the growth via the demographic changes in labor market, hence we call this set of variables as "demography parameters." The change of investment rate affects growth via the capital accumulation process and we call this an "investment parameter." From this perspective, we can use the LTGM in order to evaluate the roles of different kinds of growth sources as follows. First, we simulate Korea's GDP per capita from the neoclassical growth model in Section 2 by calibrating the six parameters varying over time as in the data, and consider this as the benchmark simulation. We label this version of simulation as "Simul." Second, we simulate by fixing all six policy parameters by their time-invariant long-run averages, i.e., by the 1960-2014 period annual average growth rates of productivity, human capital, population, and by the 1960-2014 period average values of investment rate, working-age population rate, and labor force participation rate. We label this version of simulation as "Average," which will capture the long-run growth effects in the sense that this simulation does not allow the time-varying patterns of the growth parameters. For this "Average" simulation, the six parameters are set by  $g_{A_L} = 1.9\%$ ,  $g_h = 1.5\%$ ,  $g_N = 1.3\%$ ,  $S_{W,0} = 0.65, S_{E,0} = 0.61$ , and  $\gamma_0 = 0.32$ . Figure 10 compares these two sets of simulations with the actual data. The full simulation, "Simul", captures the growth path of the actual real GDP per capita very well. The gap between the actual data and the "Simul" is due to the differences in the capital accumulation between the PWT 9.0 data ("rkna" variable) and the simulated capital stock which is constructed as in the law of motion equation (2) of the model. The "rkna" in the PWT 9.0 data is constructed by differentiating the capital goods into four kinds of assets: structures (including residential and non-residential), machinery (including computers, communication equipment and other machinery), transport equipment and other assets (including software, other intellectual property products, and cultivated assets), with applying different depreciation rates and relative prices, while we apply the common average depreciation rate and no relative price changes in our simulation.<sup>13</sup> Thus, the gap between the "Actual" and the "Simul" represents the compositional changes of heterogeneous types of capital assets

<sup>&</sup>lt;sup>13</sup>See Feenstra, Inklaar, Timmer (2015) and User Guide of PWT 9.0 for more detailed discussion about the capital construction of the PWT 9.0 data.

over time in the process of Korean economic growth. It is interesting to notice that there are virtually no gap until the mid-1980s and the gap started to emerge only after 1985 and gradually widened afterward. This implies that the compositional changes in aggregate capital seems to matter only after the mid-1980s. The comparison of the two simulations of "Simul" and "Average" in Figure 10 reveals another interesting feature of Korea's growth process. The "Average" represents mainly the long-run average growth effect holding the labor market demography and investment rates fixed. The simulated real GDP per capita of the "Average" simulation is higher than that of the "Simul" simulation for the 1960-1985 period, which is revered for the 1985-2014 period. This shows that the transitional growth factors such as improvements of the labor force participation and investment rate played a substantial role in Korea's growth.

Third, to isolate the effect of the demographic factors, we simulate Korea's GDP per capita by changing only the working-age population share, labor force participation rate, and population growth rate according to the quartic-polynomial-fit trends, and fixing the rest of variables at the long-run average values. Similarly, we can isolate the effect of investment promotion by allowing the time-varying investment rate only. The combined time-varying effect of both demography and investment parameters can be inferred by similar method also. The simulations labeled as "Demography," "Investment," and "Both" in Figure 11 represent such effects, respectively. It is interesting to notice that using the nonlinear trends of labor market demography and investment parameters, the model (simulation "Both") can fit the data very well, even though we fix the "fundamental parameters" of human capital growth rate and productivity growth rate. In this sense, the LTGM is a promising tool to predict what would happen in response to the changes of labor market and investment policies and environments, with the appropriate selection of the long-run growth rates of productivity and human capital.

Fourth, the tight-fit of the model simulation to Korean economic growth by the time-varying labor market demography and investment parameters does not imply that the main engine of Korea's growth is such changes in transitional growth parameters. Such fitting performance is based on the productivity and human capital growth rates of 1.9% and 1.5% every year in the background. To evaluate the role of such fundamental parameters in the LTGM, we simulate the model at the time-varying labor market demography and investment parameters, but turning

Figure 10: Comparison of Predictions from Fully Time-varying and Average Constant Simulations



Note (1) Each line represents the actual or the predicted path of GDP per capita using different calibration methods.

Note (2) "Actual": Actual GDP per capita, "Simul": Predicted GDP per capita calibrating at fully time-varying parameters, "Average": Predicted GDP per capita calibrating at constant parameters of average values during the sample period.

Figure 11: Labor and Investment Policy Effects



Note (1) Each line represents the actual or the predicted path of GDP per capita using different calibration methods.

Note (2) "Simul": Predicted GDP per capita calibrating at fully time-varying parameters, "Average": Predicted GDP per capita calibrating at constant parameters of average values during the sample period, "Demography": Predicted GDP per capita allowing time-variation only for the labor market demography parameters, "Investment": Predicted GDP per capita allowing time-variation only for the investment rate parameter, "Both": Predicted GDP per capita allowing time-variation for both labor market demography and investment rate parameters.

off the productivity growth, human capital growth, or both to zero. The simulated paths of the real GDP per capita of these simulations, are labeled as "No g\_h," "No g\_A," and "Neither," respectively, in Figure 12. This shows that Korea's growth would have been much lower if the promotion of investment and labor market demographic factors had been the only sources of growth.

In the year of 2014, Korea's real GDP per capita is \$34,300 in 2011 USD using national prices and \$35,103 using PPP adjusted prices and population data in PWT 9.0. The Korea's PPP-adjusted real GDP per capita in 2014 is slightly lower than that of Japan (\$35,358) and a little higher than that of Spain (\$33,864) in the same year. In 1960, Korea's PPP-adjusted real GDP per capita was \$1,175 which was lower than those of Kenya, Tanzania, Bangladesh and Haiti, while those of Japan and Spain were \$5,351 and \$5,741. Without human capital growth, Korea's 2014 real income level would have been \$14,597 (close to level of Brazil in 2014). Without productivity growth, Korea's 2014 real income level would have been \$12,178 (close to level of South Africa in 2014). With neither of productivity and human capital growth, Korea's 2014 real income level would have been \$5.970 (close to level of Bolivia in 2014). The above comparison clearly illustrates that main backbones of Korea's "miraculous growth," as is asserted by Lucas (1993), are the productivity growth and human capital accumulation, although demographic changes in labor market and investment promotion also played an important role. That is, Korea's growth experience shows that for successful and sustainable growth, the most critical factors are productivity and human capital growth, i.e., the fundamental sources of long-run growth rather than the sources of transitional growth, as most of the growth models assert.

## 5 Conclusion

Korea's remarkable growth experience itself may inspire the developing world because Korea started such development from a comprehensive set of adverse conditions (colonization, massive civil war, corruption, a lack of physical and human resources, political instability and incessant ideological conflicts etc.) that are often mentioned as critical barriers to development among current developing countries. However, without knowing what is actually behind such a growth

Figure 12: Long-run Growth Effects





Note (2) "Simul": Predicted GDP per capita calibrating at fully time-varying parameters, "Both": Predicted GDP per capita calibrating at constant fundamental parameters of human capital and labor-augmenting productivity growth, "No g\_h": Predicted GDP per capita with no human capital growth, "No g\_A": Predicted GDP per capita with no labor-augmenting productivity growth, "Neither": Predicted GDP per capita with neither human capital nor labor-augmenting productivity growth. process, Korea's development experience would be useless for other developing countries. This paper attempted to provide such knowledge to shed light on the underlying mechanisms of Korea's growth from a macroeconomic perspective using the framework of the neoclassical growth model, which is the workhorse of the World Bank's LTGM project. From a counterfactual decomposition analysis, we found that the most important source of Korean economic growth for the 1960-2014 period was productivity growth, contributing to the growth of GDP per capita by 1.9% each year on average for 55 years. The second largest contributing component was human capital accumulation (1.5% each year), and the capital deepening effect was the third (1.3% each year). Demographic compositional changes such as the increases in the working-age population share and labor force participation rate also contributed to GDP per capita growth substantially by 1% each year. These results show that the underlying sources of Korea's growth were fairly balanced among different growth components, and productivity growth was the main driving force behind the scenes. This picture is different from what many of the first generation of Korea's development policy makers used to have in mind, who considered human or physical capital accumulation as the main engine of Korean growth. In fact, that was the case in the 1960s and 1970s. In the 1960s, human capital growth, based on rapid expansion of universal education at primary and secondary levels of schooling, was the main engine of Korea's growth. In the 1970s, capital deepening due to the increasing investment rate promoted by export-oriented industrial policies indeed was the main engine of Korea's growth. However, what bolstered Korea's sustained growth throughout, particularly for the 1980-2010 period, was productivity growth, which has been rarely emphasized in most discourse about Korean economic growth. We characterized the important features of the LTGM as a simulated prediction or policy prescription tool, by applying the model to Korea's growth experience expost. We found that the model under or over-predicts the growth performance when the economy is in transition, but the model is calibrated from the steady-state cum status-quo approach, i.e., using constant growth rate assumptions. This result itself may not be a surprise. The contribution of this paper, however, is that we could quantify how big the discrepancy could be, and also show that the fit of the model becomes very good when the labor and investment policy parameters are stabilized. The latter finding is a (pleasant) surprise because the model is not built to fit the data in a reducedform way. We also found that the model fits the data very well when the time-varying short-run growth policies such as labor market demography and investment policies are embedded into the model by nonlinear trends, together with calibrating the long-run growth policy parameters such as the growth rates of productivity and human capital as constant numbers. This finding suggests that we do not need to calibrate all variables as time-varying processes for the LTGM to predict future growth in response to changes in targeted policies such as raising the investment rate or promoting female labor force participation, conditional on fixed values of productivity and/or human capital growth rates.

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